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COMPOSITE ELECTRODYNAMIC LINER

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A multilayer liner made of a composition of two materials with different conductivities is considered. The fractions of the components in the layers are varied so that the effective conductivity increases in the direction of magnetic-field diffusion under a special law that allows one to obtain an analytical solution. It is shown for a particular example that using a temperature criterion taking into account the density, heat capacity, and thermal conductivity of the components, it is possible to produce a liner for which the magnetic-field amplitude can be increased by 30% *and the velocity and energy can be increased by a factor of* 1.6 *and* 2.7*, respectively, compared to the original version of a homogeneous metal liner.*

Introduction. Electrodynamic accelerators in which a projectile is accelerated by the pressure of a pulsed magnetic field have been designed for more than forty years. The history of the design of magnetic-field concentrators, known as magnetocumulative generators (MC-generators) of the MK-1 and MK-2 types, and their operating principles are covered in [1, pp. 65–90].

A detailed review of theoretical studies and developments of MK-generators of different types is given in [2]. Results of the research performed over forty years in different countries in the field of generation and application of ultrahigh pulsed magnetic fields are summarized in a review [3].

As noted in [1], much attention has been given to the use of MK-2-type systems for acceleration of metal bodies with cosmic velocities. In particular, it was reported that an aluminum ring with a weight of approximately 2 g was accelerated to a velocity of about 100 km/sec, although the ring evaporated in this case.

In both cases — compression of an initial magnetic field in MK-1-type systems and acceleration of macroparticles by magnetic-field pressure — important factors influencing the results are melting and evaporation of the liner, which are due to the concentration of the magnetic field and conduction currents near the liner edge that is in contact with the magnetic field.

A method of slowing down the process of heating of the liner to a critical temperature is the precooling of the liner to the liquid-hydrogen temperature 15 K [4]. More accurate account of the various factors influencing the motion of the liner requires the use of a computer [5, 6]. Comparison of calculations with experiments was made in a number of papers (see, for example, [7, 8]). The issues of the choice of optimal liner parameters are considered in [9]. In particular, according to [9], replacing the liner material, for example, copper by aluminum, it is possible to increase the projectile velocity by factor of two, and with replacement of aluminum by beryllium, the velocity increases by a factor of 1.5.

Heating on the boundary can be by lowered by increasing the skin-layer thickness, for which the conductivity of the liner material should increase in the direction of magnetic-field diffusion. Analytical solutions of model problems with conductivity depending on the coordinate by a particular law are given in [10–12]. It is shown that the Joule heat released on the boundary decreases in this case. However, for the beginning of ablation, the determining parameter is temperature and a decrease in conductivity can be accompanied by a decrease in heat capacity, for example, when a porous metal is used. Therefore, in considering applied problems, it is necessary to allow for the other characteristics of the liner material: density, heat capacity,

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and thermal conductivity. With allowance for these factors, Stankevich and Shvetsov [13] determined, using a numerical method, the maximum permissible liner velocities for temperatures below critical values (melting or volatilization points) and reported some results for a bimetal (tungsten and beryllium) liner.

1. Choice of Model and Calculation of Field Diffusion. The required variation in conductivity $\sigma(x)$ across the liner thickness can be implemented by various technological methods. We consider as an example a liner consisting of N layers with thickness h. In the ith layer ($i = 1, 2, \ldots, N$), the material with higher conductivity σ_1 has thickness a_i, and the material with lower conductivity σ_2 has thickness b_i, so that $a_i + b_i = h$. The general thickness of the liner is $L = Nh$.

The volume concentration of the first component in the ith layer is $\alpha_i = a_i/h$, and the volume concentration of the second component in the ith layer is $\beta_i = b_i/h$, where $\alpha_i + \beta_i = 1$. The x coordinate is reckoned from the left end, to which magnetic field $H_0(t)$ parallel to the plane of the liner is applied. The number of layers is N and their thickness h should be such that in passage from the ith layer to a neighboring layer, the relative variation in the concentrations α_i and β_i is rather small.

It is known [4] that the penetration of a pulsed magnetic field into the material of a liner is described by the diffusion equation

$$\frac{\partial H(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[D(x)\,\frac{\partial H(x,t)}{\partial x}\right] \tag{1.1}$$

with the boundary conditions

$$H(x=0,t) = H_0(t), \qquad H(x=L,t) = 0. \tag{1.2}$$

In Eq. (1.1), the diffusivity $D(x)$ is related to the conductivity $\sigma(x)$ by

$$D(x) = A/\sigma(x), \tag{1.3}$$

where A is a constant. As noted in [4], the magnetic properties of liner materials can be ignored since high magnetic field, far exceeding saturation fields, are considered. In the Gaussian system of units, $A = c^2/(4\pi)$, where c is the velocity of light. The volumetric density of Joulean losses in unit time is

$$w(x,t) = \frac{1}{4\pi}\,D(x)\left(\frac{\partial H}{\partial x}\right)^2. \tag{1.4}$$

The nonuniform Joulean heat release across the liner thickness gives rise to heat flows and the temperature levels of. The rate of this process depends on the thermal diffusivity χ. The reference data of [14] show that for materials such as copper, aluminum, and beryllium the following condition is satisfied:

$$\chi/D < 10^{-2}. \tag{1.5}$$

During diffusion of the magnetic field through the liner, heat exchange takes place between the points separated by a distance of the order of $l = L\sqrt{\chi/D}$. With allowance for (1.5), we obtain $l/L < 0.1$. This means that accumulation of Joulean heat $Q(x,t)$ at each point of the liner proceeds adiabatically, and, hence, it can be calculated by integrating $w(x,t)$ from (1.4) with respect to time:

$$Q(x,t) = \int\limits_{-\infty}^{t} w(x,t)\,dt. \tag{1.6}$$

As noted in [4], most of the phenomena related to magnetic-field diffusion are little sensitive to the pulse shape $H_0(t)$ in the first boundary condition (1.2). In view of this, the calculations can be simplified by considering the exponentially increasing field

$$H_0(t) = H\exp(t/T), \tag{1.7}$$

where H is a constant and T is the effective time, equal with accuracy to a factor of the order of unity to the time of increase in the field on the boundary of the liner. According to [4], nearly exponential growth of the field is observed in MK-1-type facilities, which use the magnetic-flux compression principle.

Condition (1.7) allows one to write the solution of Eq. (1.1) in the form

$$H(x,t) = H_0(t)f(x), \tag{1.8}$$

where the function $f(x)$ is a solution of the equation

$$\frac{d}{dx}\left[D(x)\,\frac{df(x)}{dx}\right] - \frac{1}{T}\,f(x) = 0 \tag{1.9}$$

subject to the boundary conditions

$$f(x=0) = 1, \qquad f(x=L) = 0. \tag{1.10}$$

We consider an ordinary case where the conductivity of the liner is constant across the thickness:

$$\sigma(x) = \text{const} = \sigma_0, \qquad D(x) = \text{const} = A/\sigma_0 = D_0. \tag{1.11}$$

The time of diffusion through this liner is defined by

$$\tau_0 = L^2/D_0. \tag{1.12}$$

A solution of Eq. (1.9) subject to conditions (1.10), (1.11) is

$$f_0(x) = \sinh[\eta\Big(1 - x/L)]/\sinh\eta, \qquad \eta = \sqrt{p_0}, \qquad p_0 = \tau_0/T. \tag{1.13}$$

Substitution of (1.13) into (1.8) and (1.4) in the case of a homogeneous liner gives the following formula for the density of Joulean losses:

$$w_0(x,t) = 1/(4\pi T)\,[H_0(t)]^2\{\cosh[\eta(1 - x/L)]/\sinh\eta\}^2. \tag{1.14}$$

This function decreases monotonically with increase in x, and, hence, the degree of nonuniformity of the heating is given by the relation

$$M_0 = w_0(x=0,t)/w_0(x=L,t) = (\cosh\eta)^2. \tag{1.15}$$

The ability of a liner to confine a magnetic field can be characterized by the magnetic flux Φ that passes through the right boundary of the liner $x = L$:

$$\Phi(t) = -D(x=L)\int\limits_{-\infty}^{t} \left.\frac{\partial H(x,t)}{\partial x}\right|_{x=L} dt. \tag{1.16}$$

By analogy with [4], we define the dimension of the skin layer by the formula

$$\Phi(t) = H_0(t)S. \tag{1.17}$$

Using (1.13) and (1.16), for a homogeneous liner, we obtain

$$S_0/L = 1/(\eta\sinh\eta). \tag{1.18}$$

The effectiveness of a liner whose conductivity $\sigma(x)$ increases with increase in x can be shown by considering a liner of the same thickness L and assuming that

$$\sigma(x) = \sigma(1 - kx/L)^{-2}, \quad D(x) = D(0)(1 - kx/L)^2, \quad D(0) = A/\sigma, \quad 0 < k < 1. \tag{1.19}$$

The case of a homogeneous liner is obtained here at $k = 0$. A solution is sought in the form (1.8), and the function $f(x)$ satisfies the equation

$$\frac{d}{dx}\left[\left(1 - k\,\frac{x}{L}\right)^2\frac{df(x)}{dx}\right] - \frac{p}{L^2}\,f(x) = 0, \tag{1.20}$$

where, by analogy with (1.12) and (1.13), we introduce the notation

$$\tau = L^2/D(0), \qquad p = \tau/T. \tag{1.21}$$

Equation (1.20) subject to conditions (1.10) has a solution

$$f(x) = y^{\lambda_1}[1-(1-k)^{\mu}]^{-1} + y^{\lambda_2}[1-(1-k)^{-\mu}]^{-1}, \quad y = 1 - k\frac{x}{L}, \quad \lambda_1 = (\mu-1)/2,$$
$$\lambda_2 = -(\mu+1)/2, \quad \mu = \sqrt{1+4p/k^2}. \tag{1.22}$$

Substituting (1.22) into (1.4), we obtain the density of Joulean losses for a composite liner:

$$w(x,t) = \frac{1}{4\pi T}[H_0(t)]^2\frac{k^2}{p}[\varphi(x)]^2[1-(1-k)^{\mu}]^{-2}, \quad \varphi(x) = \lambda_1 y^{\lambda_1} - \lambda_2 y^{\lambda_2}(1-k)^{\mu}. \tag{1.23}$$

By analogy with (1.15), we write the relation

$$M = w(0,t)/w(L,t) = [\varphi(0)/\varphi(L)]^{-2} = \frac{1}{\mu^2}[\lambda_1(1-k)^{-\lambda_1} - \lambda_2(1-k)^{-\lambda_2}]^2. \tag{1.24}$$

It is not difficult to verify that as $k \to 0$, formula (1.24) becomes (1.15). Substituting (1.22) into (1.16) and (1.17), we obtain the dimension S of the skin layer for a composite liner:

$$S/L = (k\mu/p)[(1-k)^{\lambda_2} - (1-k)^{\lambda_1}]^{-1}. \tag{1.25}$$

As $k \to 0$, formula (1.25) becomes (1.18).

The quantity M from (1.24) depends on the parameters p and k, which can be changed to obtain the required value of M. Using (1.23), it is possible to show that the least value of $w(x,t)$ is at one of the interior points of the segment (0, L), and the largest value is on one of the boundaries.

In a homogeneous liner, the greatest heat release occurs on the left boundary, and from (1.14) we obtain

$$w_0(x=0,t) = (1/(4\pi T))[H_0(t)]^2 U_0, \quad H_0(t) = H_0\exp(t/T), \quad U_0 = (\cosh\eta/\sinh\eta)^2. \tag{1.26}$$

Similarly, from (1.23) we have

$$w(x=0,t) = \frac{1}{4\pi T}[H(t)]^2 U, \;\; H(t) = H\exp(t/T), \;\; U = \frac{k^2}{p}[\mu/\nu + \lambda_2]^2, \;\; \nu = 1-(1-k)^{\mu}. \tag{1.27}$$

We note that in the last two formulas, $H \neq H_0$. Formula (1.6) takes the simple form

$$Q(x,t) = \frac{T}{2}w(x,t). \tag{1.28}$$

Thus, the calculation of field diffusion and heat release is completed.

2. Comparison of Two Types of Liner. We denote the specific heat of the first and second components by C_1 and C_2, respectively. The heat capacity of a composite liner at each point is evaluated from the formula

$$C(x) = C_1\alpha(x) + C_2\beta(x), \qquad \alpha(x) + \beta(x) = 1, \tag{2.1}$$

where $\alpha(x)$, $\beta(x)$ are the volume concentrations of the components. Let $\theta(x,t)$ be a local increase in temperature. Then,

$$\theta(x,t) = Q(x,t)/C(x). \tag{2.2}$$

In order that the temperature nowhere exceed the critical value determined by the less resistant component, the heating at the extreme points must be the same: $\theta(0,t) = \theta(L,t)$. For this, as follows from (1.28) and (2.2), the following equality should be satisfied:

$$w(0,t)/C(0) = w(L,t)/C(L). \tag{2.3}$$

Using (1.24), we can write condition (2.3) in the form

$$M = \gamma, \qquad \gamma = C(0)/C(L). \tag{2.4}$$

In the case considered, the components with different conductivities form alternating layers in which conduction currents flow parallel to the interface boundaries. This configuration corresponds to the parallel connection of two conductors, and, hence, the effective conductivity is evaluated from the formula

$$\sigma(x) = \sigma_1\alpha(x) + \sigma_2\beta(x) = \sigma(1 - kx/L)^{-2} \tag{2.5}$$

since the function $\sigma(x)$ should coincide with (1.19). In addition, we assume that on the right boundary there is only the first component present, i.e., $\alpha(L) = 1$, $\beta(L) = 0$, and $\sigma(L) = \sigma_1$. Since $\alpha(x) + \beta(x) = 1$, from (2.5) we obtain

$$\begin{gathered} \alpha(x) = (z^{-2} - R)(1 - R)^{-1}, \quad \beta(x) = (1 - z^{-2})(1 - R)^{-1}, \\ z = (1 - kx/L)/(1 - k), \qquad R = \sigma_2/\sigma_1 < 1. \end{gathered} \tag{2.6}$$

From (2.6) it follows that $\beta(x) > 0$ for all x; the function $\alpha(x)$ decreases as x decreases from the value $\alpha(L) = 1$ to the value $\alpha(0) = [(1-k)^2 - R]/(1 - R)$. For $\alpha(0) > 0$, the following condition should be satisfied:

$$0 < k < 1 - \sqrt{R}. \tag{2.7}$$

Let ρ_1 and ρ_2 be the densities of the components. Then, the specific masses m_1 and m_2 of the components and the specific mass m of the liner are evaluated from the formulas

$$\begin{gathered} m_1 = \rho_1 \int_0^L \alpha(x)\,dx = \rho_1 L(1 - k - R)(1 - R)^{-1}, \qquad m_2 = \rho_2 \int_0^L \beta(x)\,dx = \rho_2 Lk(1 - R)^{-1}, \\ m = m_1 + m_2 = m_0\delta, \qquad \delta = (1 - k - R + k\rho_2/\rho_1)(1 - R)^{-1}, \end{gathered} \tag{2.8}$$

where $m_0 = \rho_1 L$ is the specific mass of a homogeneous liner made of the first (more high-conducting) component. We substitute (2.6) into (2.1):

$$\begin{gathered} C(x) = C_1[(1 - C_2/C_1)z^{-2} + C_2/C_1 - R]/(1 - R), \\ \gamma = C(0)/C(L) = [(1 - C_2/C_1)(1 - k)^2 + C_2/C_1 - R]/(1 - R). \end{gathered} \tag{2.9}$$

In our case, both the composite liner and the homogeneous liners made of the material of the first component have the same thickness L. Thus, $D_0 = D(x = L) = D_1 = D(0)(1 - k)^2$, i.e., $D(0) = D_0/(1 - k)^2$, and the parameters p_0 and p are related by

$$p_0 = p/(1 - k)^2. \tag{2.10}$$

In addition, we assume that the critical temperature is determined by the first component, so that for both liners, the critical temperature is the same. Under these assumptions, we obtain the ratio of the fields $H_0(t)$ and $H(t)$ from (1.26) and (1.27), respectively, at which both liners at points with highest magnetic-field strength have identical heating. According to (1.26) and (1.28), for the homogeneous liner, the heating on the left boundary is

$$\theta_0(x = 0, t) = \frac{1}{8\pi}[H_0(t)]^2 U_0/C_1. \tag{2.11}$$

Similarly, for the composite liner, from (1.27) and (1.28) we have

$$\theta(x = 0, t) = \frac{1}{8\pi}[H(t)]^2 U/C(0). \tag{2.12}$$

By virtue of the above assumptions, the left sides of Eqs. (2.11) and (2.12) should be equal. Hence, using the designation of γ from (2.9), we obtain $\xi = [H(t)]^2/[H_0(t)]^2 = \gamma U_0/U$. The magnetic-field pressure on the liner is proportional to the square of the magnetic-field strength. Since in formulas (1.26) and (1.27), the parameter T is the same, the times of magnetic-field action on both liners can also be considered identical. Under these assumptions, the ratio of the velocities of the liners v and v_0 is obtained from the condition $mv/(m_0v_0) = [H(t)]^2/[H_0(t)]^2 = \xi$, i.e., with allowance for (2.8), we have $v/v_0 = \xi m_0/m = \xi/\delta$. The ratio of the kinetic energies of the liners E and E_0 is given by the formula $E/E_0 = mv^2/(m_0v_0^2) = \xi^2/\delta$.

As an example, we consider a liner made of a composition of aluminum and mica, whose parameters are denoted by subscripts 1 and 2, respectively. In this liner, the thermal stability is determined by aluminum. The constants of the materials necessary for the calculation are taken from [14]:

	Aluminum	Mica
Density, g/cm^3	$\rho_1 = 2.7$	$\rho_2 = 2.8$
Heat capacity, $J/(cm^3 \cdot deg)$	$C_1 = 2.5$	$C_2 = 2.4$
Resistivity, $\Omega \cdot cm$	$r_1 = 2.7 \cdot 10^{-6}$	$r_2 = 10^{10}$
Diffusivity, cm^2/sec	$D_1 = 200$	$D_2 = 8 \cdot 10^{17}$
Thermal diffusivity, cm^2/sec	$\chi_1 = 0.9$	$\chi_2 = 2 \cdot 10^{-3}$

The diffusivity is expressed in terms of the resistivity r as $D = 10^9/(4\pi)r$.

The number of layers is N and their thickness h should be selected so that the time of heat exchange t_Q between the two components within a layer is shorter than the length of the field pulse t_H. We denote the smaller thermal diffusivity by χ. Then, $t_Q = h^2/\chi$ in the order of magnitude. The thickness of the layer is selected according to the inequality $t_Q \leqslant t_H$, i.e.,

$$h \leqslant \sqrt{\chi t_H}. \tag{2.13}$$

For the above constants of the materials, $\chi = \chi_2 = 2 \cdot 10^{-3}$ cm^2/sec. Substituting into (2.13) the value $t_H = 10^{-2}$ sec, which is typical of many facilities, we obtain $h \leqslant 45$ μm. In formula (1.19), we set $k = 0.7$. From (2.9) and (2.8), we have $\gamma = 0.9636$ and $\delta = 1.028$. From (2.4) and (1.24), using the trial-and-error method, we find that $p = 0.8017$, which, according to (2.10), corresponds to a value $p_0 = 8.9077$. Then, from the formulas given above, we obtain $U = 0.5843$, $S/L = 0.2609$, $U_0 = 1.0103$, $S_0/L = 0.0340$, $S/S_0 = 7.6735$, $\xi = 1.6601$, $H/H_0 = \sqrt{\xi} = 1.2908$, $v/v_0 = 1.6207$, and $E/E_0 = 2.6838$.

In the present example, using the composite liner, it is possible to increase the magnetic field by 30% without exceeding the permissible temperature. The velocity of the liner thus increases by a factor of 1.6, and the energy increases by a factor of 2.7. When the composite liner is used instead of the homogeneous liner, the dimension of the skin layer, defined by formula (1.16), increases by a factor of 7.7. Usually, the region occupied by the accelerating magnetic field usually far exceeds the liner thickness, and, hence, the magnetic-flux losses in both cases can be considered negligible. However, in some situations, for example, in considering MK-1-type multistage setups [2, pp. 226], it is necessary to ensure passage of some part of the magnetic flux during acceleration of the liner for subsequent magnetic-flux compression by the accelerated liner.

A patent from the Russian Federation, No. 2107985, has been awarded received for the composite liner described above.

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